ANDREW DAVIDSON & Co., INC. the Pipeline

Valuation Commentary

First Steps in Credit OAS (part I)

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With the LoanDynamics™ Model (LDM) out the door (http://www.ad-co.com/credit_product_line/loan_dynamics_model.htm), AD&Co is at full steam working on its most needed application – incorporating LDM into our valuation/OAS system. "Credit OAS" is a natural jargon, perhaps very inaccurate, but serving its point: everyone understands what it means. We talk about an OAS model with a full generation of random loan losses and deals' credit triggers. Capable of generating prepayments and defaults concurrently, coupled with our term structure models and home price index (HPI) model, the Credit OAS system should assess value and risk of sub-prime loans and structured deals in a fundamentally sound way. Our development ideas and first results are discussed in this article; read more on this topic in next month's commentary.

Two-factor world, many states

Consideration of defaults and losses involves home prices (HP). Prepayment is a call option on a borrower's loan. Default is a put option letting the borrower exchange his/her home for the outstanding loan balance. Both interest rates and home prices are factors in this decision. Technically, both prepayment and default are American-exercise options, but, respecting irrationality in individual decisions, we model them as a sequence of European payoffs.

Before we ask the LDM to forecast prepayments and defaults, we must tell it what will happen with interest rates and home prices. The interest rate dynamic is generated by AD&Co's term structure library that is free of arbitrage and tied to swap rates and the swaption volatility matrix. The HPI dynamic was described in our April 2006 Pipeline article (http://www.ad-co.com/newsletter/Issues06/Apr06.htm), and presented in greater detail at the 2006 AD&Co conference (http://www.ad-co.com/secure/2006%20ADCO%20Presentations/HPI%20Model.pdf). Below is a summary:

- US HPI growth rate is generated by a dynamic stochastic model that includes 3 main components: the interest rate component pointing to the housing affordability, "diffusion" associated with other economic factors, and "jumps" in returns of the housing stock.

- The Kalman filter is employed to separate these components historically as well as to identify the model's optimal parameters. When applied retrospectively, the filter is able to estimate today's value for (i.e. "initialize") the diffusion term.

- AD&Co simulator preserves correct scales and orthogonality (mutual and serial independence) of random shocks that disturb the interest rates, the HPI diffusion, and the HPI jumps.

Modeling sub-prime deals backed by geographically dispersed collateral may require constructing geographical HPI models for states and even MSAs. We have reviewed several practical methods of achieving this goal. One of them, the "Alpha-Beta" approach, views the relationship between states and the US in a way resembling stock-to-an-index. It captures well the regional exposure to the US home prices, but leaves the question of modeling states having large idiosyncratic volatility open.

The other method, Principal Component Analysis (PCA), traces geographical home prices to 1-2 additional random variables that can be extracted mathematically. Our analysis has shown, after the US HPI index is accounted for, the second component can be interpreted as "economic swing", the difference between dense economic centers and areas with available land.

Valuation methods

Regardless of the HPI generation process, the LDM model has two additional state variables: the delinquent and severely delinquent populations of borrowers (or, for a single loan, their probabilities). Therefore, we will be dealing with a large number of random variables thereby reducing the valuation choice to the Monte-Carlo method, even if the instrument in question is an unstructured loan or a pool of loans. Although Monte-Carlo is a relatively slow option, its convergence is not strongly affected by the number of variables. We expect to face several challenges when running Monte-Carlo for the credit OAS application:

- Credit models work better on loan-level data. When it's available, a large number of loans in a pool can make computations slow.

- Senior classes in sub-prime structures are often well protected and start eroding under large and rare declines in home prices. It may take too many paths to run before these declines get simulated statistically.

- Sub-prime deals are usually notoriously slow in processing. This is a known observation made well before consideration of passing the credit data to Intex.

It is therefore important to use the most efficient sampling when valuing sub-prime deals with LDM. I will describe below a pair of methods in development; their fundamentals are described in Glasserman (2003).

Special Monte-Carlo 1: control variate/path weighing

Control variate correction is a popular method allowing for the estimation and correction of the valuation error using another financial instrument. This method involves two instruments, A (instrument in question) and B (control variate); an exact price of the latter is assumed to be known.

Below is Glasserman's description:

$$P_A \approx \frac{1}{n}\sum_i P_A^{MC_i} + \beta(P_B - P_B^{MC_i})$$

Note two features:

- The method is applied path-wise (MC_i denotes *i*-th path's value)
- Beta is the optimal "regression" coefficient.

In essence, we present-value instruments A and B path-by-path, store path-wise values and employ them not only for averaging but also for regressing versus each other. As a result, Monte-Carlo values for A get effectively re-weighted.

Although this method is not free of problems, it's designed to always reduce valuation error. It does not matter if Beta is positive or negative, small or large. Correlation between asset A and control variate B is what matters most. This method is easily extendible to the case with many control variates. We have tested it for OAS valuation of MBS using a set of swaptions as control variates.

A tempting way to apply the control variate method to the Credit OAS problem is to introduce artificial home price "options", such as a set of mathematical puts, $E(K - HP)^+$. The exact value of this expectation can be computed analytically because the HP is lognormally distributed in the AD&Co model, with a known mean and variance. For suitably chosen "strikes" *K* of the puts, one can replicate the work of the default option thereby capturing the most important statistical property of the random home price process.

Special Monte-Carlo 2: importance sampling

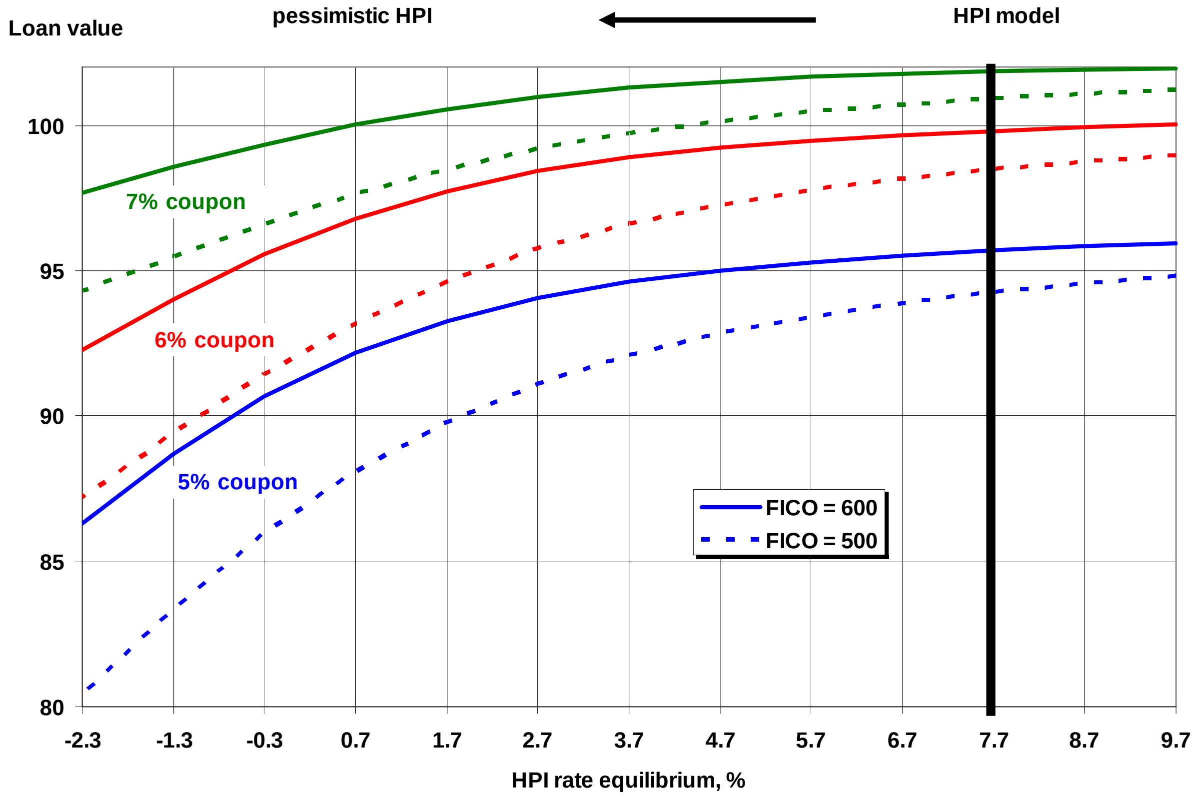
If the default option is well out-of-the-money (i.e. home prices would have to take an unusual dive to trigger any losses), it may take too many Monte-Carlo paths to produce statistically suitable valuation results. A common recipe here would be using the importance sampling Monte-Carlo. Instead of sampling home prices to their objective probability distribution, we can create a "pessimistic" distribution forcing most HPI paths to go down. Each path will yield a certain present-value that will be weighted by the ratio of two density functions, the objective one and the artificial one. A common example of importance sampling found in books is pricing deep out-of-the money options.

It's easy to see that, while these pessimistic HPI paths will be important for simulating the defaults and losses, they won't have high weights. The paths that are more likely to occur under the objective distribution will have high weights, but won't produce high defaults or losses. This method has to be used with caution, however. Recall that an MBS, even without losses, still prepays randomly. This means that, with a credit-importance sampling, few high-weight paths may distort the overall value of the MBS. We think of using the importance sampling method solely for valuing the losses. This may increase the processing time somewhat because the loss-free Monte-Carlo will have to be run separately.

Some results

Exhibit 1 depicts the loan value dependence on the HPI user-defined equilibrium shift. The 7.7% level matches the model's long-term HPI growth given today's level of interest rates, reduced levels are pessimistic, inflated levels are optimistic with each unit corresponding to 1% of the HPI growth rate. Aside from this shift in equilibrium, the OAS model operated with stochastic interest rates and US HPI rates using 10,000 quasi Monte-Carlo paths; the OAS level was set to 0 in all tests (recall that we simulate losses explicitly and do not need to reflect them in the pricing spread).

Exhibit 1. OAS valuation results for hypothetical sub-prime loans by rate and FICO



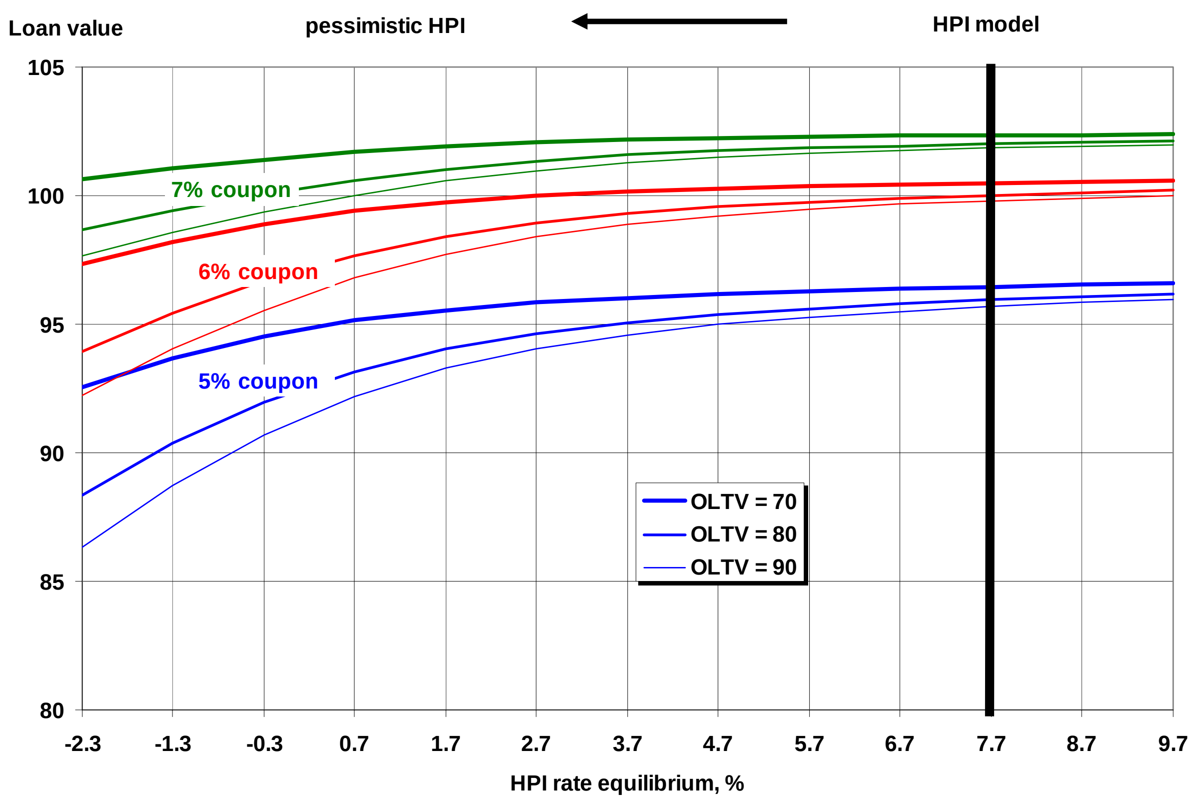
All loans in this analysis were assumed new, 100% current, 90% LTV, $150K size, with discount, par, and premium coupons. The analysis was performed using zero tuning of the "SATO residual" factor to mute possible logical contradictions in assumptions (such as a large SATO difference between similar loans) and to enable this purely illustrative exercise.

As expected, the curves plotted in Exhibit 1 are concave due to the presence of the short default option position. This very fact proves the necessity of a stochastic HPI modeling; had we assumed static HPI rates, we would have understated average losses. It is also seen that the loss of value will be leveraged in low-WAC, slow-paying, pools. This key fact is related to both a higher time value of the default option and a longer stream of losses (think of an IO).

Valuation effect of a low credit score seems to be interesting and is in line with the LDM modeling vision. Even with benign home prices, low FICO borrowers tend to miss payments and, ultimately, default subject to a minimum loss severity. This trend is related to income and other factors affecting credit scores. With a drop in home values, the same process causes larger losses for investors.

Exhibit 2 shows valuation results by original LTV using a FICO of 600. The most notable fact resulting from the LDM and depicted in this exhibit is the high sensitivity to OLTV below 80%. With this standard OLTV requirement passed, no mortgage insurance or second loan needed, the loss picture gets improved drastically.

Exhibit 2. OAS valuation results for hypothetical sub-prime loans by rate and OLTV



References

Glasserman, P. *Monte-Carlo Method in Financial Engineering*, Springer, 2003.