The Concept of Credit OAS in Valuation of MBS

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With the growth of the non-agency mortgage market and the current focus on subprime mortgage losses, finding fair economic value in credit sensitive mortgage-backed securities (MBS) is now a crucial ingredient in the functioning of the capital markets. About a decade ago, credit modeling first garnered the attention of the academic world with applications to corporate liabilities when leading researchers, known for their work in asset pricing, options, and interest rates, switched their focus to modeling credit events (see Duffie and Garleanu [2001]).

Development of MBS modeling has traditionally been delegated, with few exceptions, to practitioners. Mortgage modeling generally involves both theoretical and empirical analysis because borrower behavior cannot be determined by theoretical considerations alone. Modeling defaults and losses in mortgage pools demands more data than just forecasts of prepayments and is most accurately predicted at the loan level.

Until recently, the primary holders of credit risk in the mortgage market were the government-sponsored enterprises (GSE) Fannie Mae and Freddie Mac, depository institutions through their lending and portfolio activities, and mortgage insurance companies. Credit modeling was performed at these institutions and at the major rating agencies. The credit models, with the exception of those of the GSEs and mortgage insurance companies, generally focused on setting loan loss reserves and capital levels rather than the fair value of credit sensitive instruments. In recent years, however, several major Wall Street firms and independent analytical firms have built credit valuation models.

The MBS investor, the risk-bearing party in the non-agency securities market, has had to rely almost entirely on external modeling efforts. For example, an AA-rated MBS tranche investor would employ the discount spread commonly used in the AA fixed-income market and apply it to otherwise loss-free cash flows of the tranche. The ongoing 2007 mortgage crisis highlights the problem of overreliance on credit-rating labels and spreads. It appears that AA and AAA ratings no longer offer the level of protection they once did. Even if a reasonable expectation of losses does not trigger the write down of a senior tranche, further deterioration in market conditions might. Davidson [2007] describes the mechanics and flaws of mortgage origination, underwriting, and investment as well as the roles of all parties involved in the crisis.

Two important points should be raised. First, the investor has to do his homework by obtaining the collateral data, forecasting delinquencies, defaults, and losses, and deriving economic values. Second, a single baseline forecast does not earn an "A"; the MBS and asset-backed securities (ABS) markets value tranches by considering all possible, even perhaps remote, losses. We discuss this aspect in the next section and show why a stochastic model is necessary to explain market prices.

THE NECESSITY OF STOCHASTIC MODELING OF LOSSES

In this section, we provide a few examples from the mortgage industry that illustrate why the entire probability distribution of losses is required in order to find the economic value of financial instruments.

Default Option Modeling

Much like a prepayment, a loan default is an option. A borrower who finds himself in a grim financial situation and unable to make his mortgage payments, can either sell his house or walk away from his mortgage. The decision depends largely on the amount of the borrower's equity in the home which is a function of the home price. A default can be viewed as a put option on the property and a refinancing as a call option on the loan. Given the optional nature of default, default rates and losses depend nonlinearly on home prices. Like the refinancing option, the default option is not exercised efficiently and its laws can best be seen through the prism of empirical modeling. Common refinancing patterns suggest negative convexity of MBS prices with respect to interest rates. We assert positive convexity of losses (negative convexity of prices) with respect to home prices.

What follows from this option-like argument is that the expected default rates and losses in a cohort of loans cannot be accurately assessed using a single forecast. Examples suggest that the understatement of loss expectation when making projections using a static framework can be large.

Asset-Backed Securities

Let us assume that an asset-backed securities (ABS) deal (backed by subprime mortgages or other insecure loans) is composed of three tranches or classes: a senior tranche, mezzanine tranche, and junior tranche. Suppose that the credit enhancement structure protects the junior tranche up to 10% of collateral loss, mezzanine tranche up to 20%, and senior tranche up to 30%. This means that a collateral loss of, say, 5% will not spread into these three classes, but will be absorbed by subordinate classes. A 15% collateral loss will considerably reduce cash flow to the junior tranche (up to its total principal write-down), but will not cause any loss in the mezzanine and senior classes. A 25% loss may cause full principal write-downs on all but the senior tranche, and so forth.

At first glance, all an investor needs to know is a single number—the projected collateral loss. The market prices of all tranches, however, are somewhat discounted for losses, albeit to various extents. No single loss scenario can explain the market values across the capital structure. This mystery disappears if we reframe the loss as a random number. Instead of assuming that the loss can be an either 5%, 15%, or 25%, we can instead assume a certain probability associated with a particular loss scenario so that each is subject to a probability distribution. This randomness can be easily attributed to both market factors (described in the next section) and with model uncertainty.

Gauthier [2003] suggests recovering the loss distribution using market prices across the credit structure. Suppose we have a grid of loss scenarios, ranked from lowest to highest. Each loss level results in a set of prices across the tranches of the ABS. Next, we determine the loss probability that best equates the weighted average price for each of tranches of the ABS with the actual market price. This method is a low cost approach because it requires no loss modeling and recovers a probability distribution of defaults and losses from directly observed market prices. As attractive as this method appears, it outsources the modeling effort and takes for granted the prevailing market view. For example, in fall 2007, ABS prices were unstable suggesting that market makers had lost firm views on future losses. The Gauthier model also requires the user to construct each scenario grid point by somewhat arbitrarily linking losses to prepayments and delinquencies, and expressing them as time-dependent vectors. Of course, the timing of prepayments and losses is also important, but they cannot be accurately predicted without modeling.

Mortgage Insurance, Guarantees, Losses, and Capital Requirements

The insurance industry requires the protection seller to maintain capital sufficient to cover all possible losses. The potential physical loss on a property thus represents only part of the insurance coverage and premium. The additional capital that must be maintained to cover losses above the anticipated level only earns the risk-free rate which then leads to the imposition of a *capital charge.* The amount of this charge depends on the loss distribution, protection depth, and desired return on capital. Even if losses above the stated coverage are highly unlikely, the capital charge will depend on the coverage. The GSEs are expected to offer the largest premium because they provide full protection against loss.[1]

A similar argument applies to other participants in the MBS market. For example, credit default swap (CDS) protection sellers are insurers of ABS tranches rather than insurers of loans. Mortgage originators that keep unsecuritized mortgage loans on their balance sheet set aside a so-called reserve for losses. In one of the following sections, we will present details to better quantify this problem. In essence, we will show how financial business logic contributes to the definition of the price of risk and, thus, to the concept of the risk neutrality of losses.

Exposure to Interest Rates

Default and loss rates cannot be considered independent of interest rates. Historical analysis points to a negative correlation between mortgage rates and the systematic component of home price appreciation (HPA). This suggests that losses in MBS pools should rise and fall with interest rates. Hence, the interest rate exposure of subprime MBS and ABS is different from the exposure established without credit modeling. The negative relationship between HPA and interest rates may look both counterintuitive and contradictory to simple statistical measurements. We discuss this topic later in the article.

Is the Loss Stream an IO or a PO?

When a loss model is replaced by "equivalent" discounting, an assumption is being made that the loss stream looks like an interest-only strip (IO). Indeed, an annual loss at a constant rate of 1% of the remaining principal is equivalent from a pricing standpoint to an additional discount spread of 100 basis points (bps). We previously mentioned that if rates change, losses would change too, so the usual option-adjusted spread (OAS) method does not provide an accurate rate exposure. However, are losses truly an IO?

If part of the collateral pool is already severely delinquent, it will likely go through foreclosure regardless of the interest rate. Hence, a loss stream of this kind resembles a principal-only strip (PO), not an IO. Furthermore, unlike a PO, the liquidation time is not driven by interest rates. Therefore, the loss stream of a severely delinquent loan looks more like a portfolio of zero-coupon bonds with a duration of 0.5–1.0 years than the typical duration range of an IO. In order to capture interest rate exposure correctly, a good model must relate the nature of losses to the delinquency composition of the collateral.

FACTORS AND INGREDIENTS OF CREDIT OAS

The concept of Credit OAS refers to an extension of the OAS method that involves an explicit model of credit events such as delinquencies, defaults, and losses and their influences on cash flow. In other words, instead of generating a loss-free cash flow for an MBS and discounting it with a purposely inflated credit spread, we delve into full credit modeling of the underlying collateral. For structured deals, the losses are translated into a tranche's write-downs and interest shortfalls. The cash flows generated this way are then discounted using a discount rate plus an OAS that is similar in level to an OAS of credit-perfect instruments.

A well-designed valuation system should employ random factors that follow risk-neutral dynamics. In our case, randomization of future events must include prepayments, defaults, losses, rate resets, and deal triggers. Specifically, market randomness leads to the probability distribution of the losses. Yet, subjective views of these probabilities by a particular empirical modeler will rarely generate accurate market prices. Empirical modeling, fair or biased, lacks the market price of risk. As in the mortgage insurance example, the expected loss is only part of the insurance premium. This leads to the problem of defining the risk-neutral conditions imposed on the factors' behavior which we will be discussing throughout this article.

Market factors employed in this modeling process include, but are not limited to, interest rates. Interpretation of the default option as a put on property suggests that a relevant home price indicator (index) must also be included. Another good candidate is the unemployment rate because borrowers who lose their income are likely to become delinquent on their loans. For example, a Deutsche Bank model presented by Patruno, Ilinic, and Zhao [2006] employs a joint simulation of these three factors. But even with a long list of relevant economic factors, and even if they are all known, the default rate and losses could only be approximately predicted. This simple example illustrates that a model's systematic error (bias) is itself an important risk factor. The market does not need to price random oscillations of outcomes around modeled forecasts because they are diversified over time. Neither should the deviations of a single loan's behavior from the average be a concern, because they are diversified in large pools. Instead, what should concern investors is the risk that a model systematically understates the default rate and losses.

In our work at Andrew Davidson and Co., Inc., we consider it most practical to use interest rates and home prices as key factors in Credit OAS. We clearly realize the need to employ a risk-neutral model for prepayments, defaults, and losses, but this goal can and should be reached via alteration of a reasonably well-designed empirical model. The argument made in our work on prepayment risk neutrality (Levin [2004] and Levin and Davidson [2005]) is that a risky return generated by any factor can be exactly replicated by altering the factor's drift (or value). Therefore, a risk-neutral transformation can be accomplished via the model's "tuning" to market prices of MBS benchmarks.

All OAS systems designed for MBS use some type of risk-neutral models of the interest rate term structure. Term structure modeling is a well-developed area of financial engineering. A good model should be calibrated to market instruments (rates and options) that are relevant to MBS funding and optionality. We focus the remainder of this section on three ingredients of Credit OAS that are required beyond the traditional OAS approach: a home price index (HPI) model, model for defaults and losses, and specifics of valuation computations.

Home Price Appreciation Model

A stochastic home price appreciation (HPA) model based on historical data from the U.S. Treasury Department's Office of Federal Housing Enterprise Oversight (OFHEO) was described in Levin [2006] and was built using a dynamic asset model as a prototype. Let us assume that a home price index (HPI) is analogous to a stock or stock index. It is random but continuous, and its return rate contains a systematic component (drift) and white noise (volatility). Hence, we begin by modeling the HPI return rate, i.e., the HPA rate. Exhibit 1 depicts visible negative correlation between the systematic part of the U.S. HPA rate published by OFHEO and a long market rate which in this case is the 10-year Treasury bond.

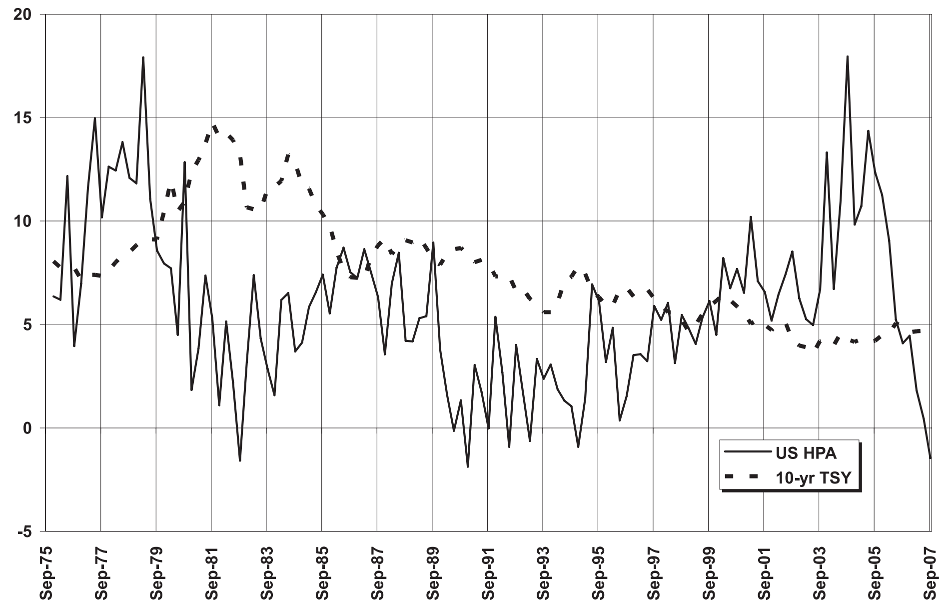
The HPA rate features volatility which we term "jumps." Note that jumps is a practical term used simply to describe white noise observed in discrete time and is mathematically unrelated to Poisson jumps with random arrival. Exhibit 1 suggests that the HPA rate is indeed analogous to a stock's return. We can postulate that the U.S. HPA rate can be generated by a dynamic stochastic model which includes three main components: the interest rate component which indicates housing affordability, diffusion which is associated with other economic factors, and jumps in the returns of the housing stock. More formally, in continuous time,

$$HPA(t) = HPA_{\infty} + \underbrace{k[R_{\infty} - R(t)]}_{\text{housing affordability}} + \underbrace{D(t)}_{\text{diffusion}} + \underbrace{\sigma_{HPA} W_{HPA}(t)}_{\text{white noise (normal jumps)}} \tag{1}$$

In Equation (1), R is the key interest rate, k and σ_{HPA} are positive constants, and HPA_{∞} and R_{∞} are the historical averages for HPA and R, respectively. For each $HPA(t)$ scenario, $HPI(t)$ is computed as $HPI(t) = HPI(0)\exp\left[\int_0^t HPA(\tau)d\tau\right]$. Equation (1) includes a random disturbance (i.e., the white noise W_{HPA}) and a continuous random process $D(t)$. The latter can be a simple single-dimensional linear mean-reverting process or, more rigorously, a two-dimensional oscillator, linear or nonlinear. An oscillatory behavior lets us view $D(t)$ as a realistic demand–supply imbalance that rises when demand exceeds supply and falls otherwise. A proper set of initial conditions has to be provided for $D(t)$. In the single-factor case, the initial condition is just $D(0)$. In the two-dimensional version, both $D(0)$ and the first derivative $\dot{D}(0)$ are necessary to initialize the process, thereby allowing us, to some extent, to capture both the formation and the bursting of a housing bubble.

We employ the Kalman filter algorithm to separate the historical jump, diffusion, and interest rate components as well as to identify the model's optimal parameters. When applied retrospectively, the filter is able to estimate today's value for (i.e., initialize) the diffusion term. Exhibit 2 shows the three components of the HPA rate in historical retrospective beginning in September 1989.[2]

EXHIBIT 1
History of OFHEO U.S. HPA Rate and 10-Year Treasury Rate



The interest rate effect which indicates the affordability of housing has been constantly improving. Because the diffusion term is mean reverting our HPI model is not a real world martingale, but it is capable of explaining the inertial effect of both deficient and excessive housing stock. To some extent, the model explains a housing price bubble (e.g., note that the diffusion increased from 2004 to 2005 and fell rapidly in 2007). However, once this happens, the mean-reverting model always begins predicting HPA recovery.

With properly selected parameters, the model captures well both overall HPA volatility and its relationship to interest rates. We also like to view both $D(0)$ and $D(\infty)$ as convenient tuning parameters as well as factors of risk; changing $D(0)$ and $D(\infty)$ by the same amount may be equivalent to shifting HPA_∞. By altering these two parameters, the short-term dynamics and the long-term level of HPA, respectively, can be changed; in particular, we can employ this method to achieve a better understanding of the risk neutrality of HPA. This approach will be employed throughout the article.

Our HPA model possesses a few interesting properties. Each HPA measurement, regardless of how often it is taken, contains a jump; hence the short-term uncertainty of HPA is σ_{HPA}. The short-term expectation is $HPA_\infty + k\,[R_\infty - (R(0)] + D(0)$; in particular, it is not equal to the previous observation which may contain the jump. Long-term HPA uncertainty includes the standard deviations of interest rates and of the diffusion term. The model purposely ignores an explicit dependence on economic factors other than interest rates. Indeed, should we decide to include these factors we would have to model them, but this task would be repetitive of introducing the unnamed forces, white noise W_{HPA} and diffusion D.

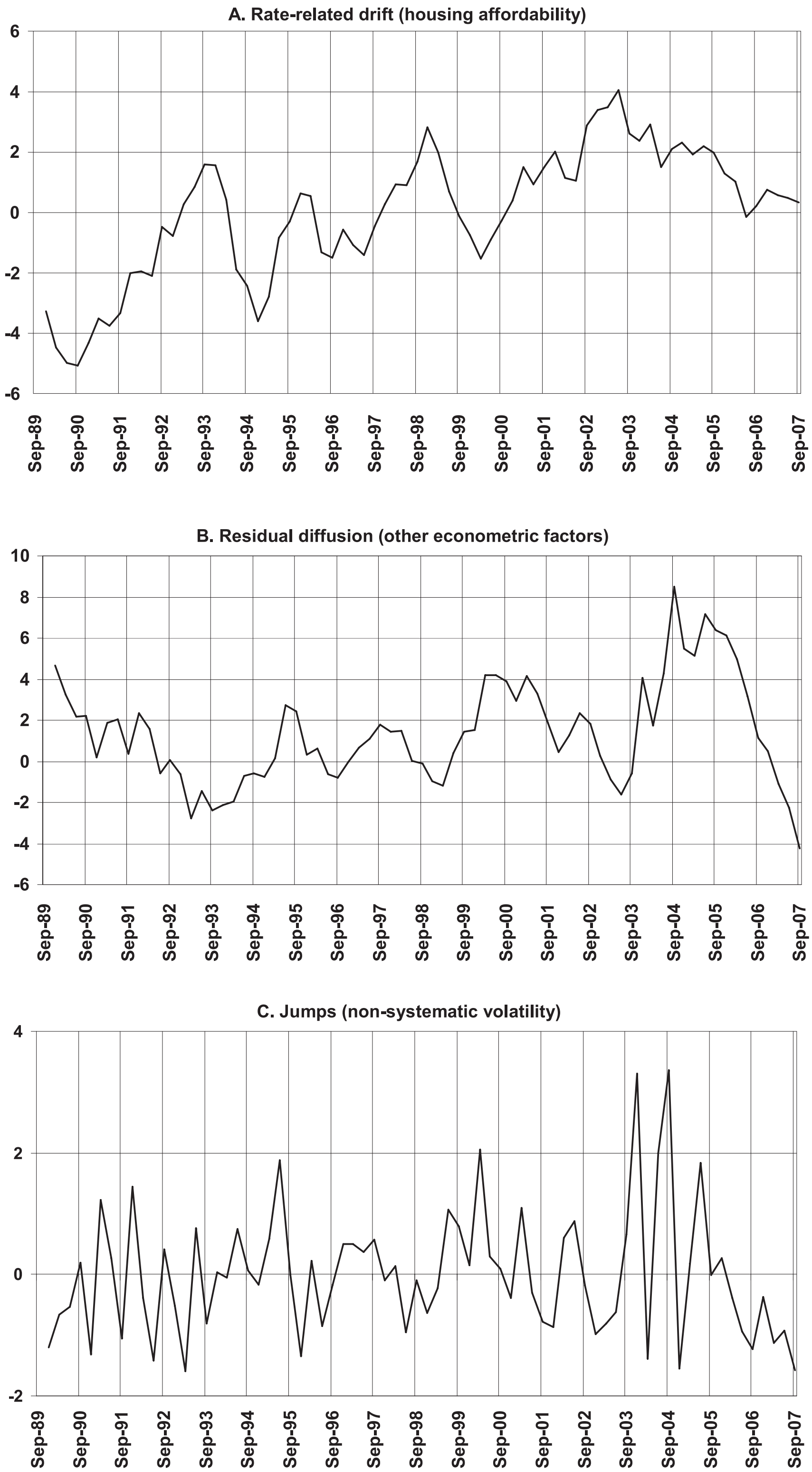
Modeling subprime deals backed by geographically dispersed collateral may require constructing geographical HPI models for states and even metropolitan statistical areas. We have reviewed several practical methods of achieving this goal. One method, the Alpha-Beta approach, views the relationship between states and the U.S. as that of stocks to an index. It captures well the regional exposure to U.S. home prices, but leaves open the question of modeling states that have large idiosyncratic volatility.

Another method, principal component analysis (PCA), traces geographical home prices to one to two additional random variables which can be extracted mathematically from the regional HPA time series. Our analysis has shown that, after accounting for the U.S. HPI, the second component can be interpreted as economic swing, or the difference between dense economic centers and areas with available land.

Modeling Defaults and Losses

Along any interest rate and HPI trajectory, we can assess a borrower's incentive to refinance and propensity to default. The refinancing option's moneyness is primarily driven by interest rates and, to some extent, by the amount of a borrower's equity, which impacts his ability to cash out. The default option and loss severity depend primarily on

EXHIBIT 2
Genesis of HPA Rate (September 1989 to September 2007)



the loan-to-value ratio (LTV), which is driven by the HPI scenario. A loan having a 95% LTV at origination becomes a 105% LTV loan if the home price drops 9.5% after origination. Once the amount of debt exceeds the market value of the asset, the homeowner may stop paying his mortgage if he finds himself in difficult financial circumstances. In this example, such an action would lead to a minimum 5% loss. A worse HPI scenario would result in a higher probability of default and a higher loss. A borrower who can refinance his mortgage loan may avoid the path to default. Hence, the default option's exercise depends also on the level of interest rates.

Refinancing and default are functions of both interest rates and home prices. Therefore, a good model should simulate them concurrently and, in particular, should consider factors affecting both decisions. Interestingly enough, it appears that, while agency MBS prepayment models traditionally revolve around averaged pool characteristics, a good prepayment model for subprime MBS requires information about LTV, credit score, loan payment status, and other loan-level details typically employed for credit modeling. At Andrew Davidson and Co., Inc., we have developed a model, the LoanDynamics™ model (LDM), which simulates concurrently the prepayment, delinquency, default, and loss severity rates along every scenario of interest rates and home prices. The LDM was named in recognition of the transitional framework employed in its design.

The LDM assumes that every loan can be "current" (C, normally paying), "delinquent" (D, 60–150 days late), "severely delinquent" (S, over 150 days late), or "terminated" (T, the exit state). The derivation of the transitional probabilities as functions of loan characteristics and market data represents the essence of the modeling approach. For example, the delinquency process (i.e., the transition from C to D) depends largely on the credit score (FICO), documentation, and spread at origination (SATO), which serves as additional evidence of a borrower's creditworthiness.[3] The simulated LTV affects the ability of delinquent loans to cure, or prepay without loss, as well as impacts the degree of loss severity. Loan characteristics such as borrower documentation, lien position, loan purpose, property type, mortgage insurance, and geographical location are additional static modeling inputs. The LDM transitions can be mapped into the Bond Market Association (BMA) standards to produce traditional measures for delinquency and default rates.

This few-state transitional model has some advantages over other methods used in MBS credit modeling. First, it is economical in size and includes processes that are directly and unambiguously observed from each loan's essential status change. In contrast, the BMA definition of defaults refers to loans that stop paying and never recover; such a process cannot be measured in real time. Second, the LDM lets us employ the known initial conditions of loan status. A seasoned pool's composition at time zero (C-D-S) is certainly an important ingredient in forecasting future delinquencies and defaults.[4] Finally, unlike commonly used standards that set hard restrictions on the time to terminate, such as 6 or 12 months, all LDM transitions (including termination) are random. While the exact time in each state is unknown, the average time is inversely related to the probability of leaving the state. For example, the average time to terminate a severely delinquent loan is equal to 1/Prob(S->T). More details about the LDM are described by Lundstedt [2007].

Setting Up Credit OAS Calculations

Once we define a risk-neutral evolution Q of interest rates and home prices along with cash flows, $CF(t)$, which are contingent upon these processes, the economic value of a T-maturity instrument becomes

$$P = E^{Q}\left[\int_0^T CF(t)\exp\{-\int_0^t [r(\tau)+crOAS]d\tau\}\,dt\right] \quad (2)$$

where *crOAS* is Credit OAS (i.e., the residual discount spread). The variable *crOAS* does not carry credit risk because we explicitly impair the cash flow; it is merely a measure of liquidity or mispricing. Calculations of economic values using *crOAS*, as well as iterating for *crOAS* using known prices, is done via Equation (2). Instruments traded at a wider *crOAS* are generally considered cheap. Exposure to interest rates and HPA rates will be measured using constant *crOAS*.

Let us transform Equation (2) through measures often associated with MBS modeling. We assume that an MBS or an ABS pays coupon $c(t)$, amortizes at a total rate of $\lambda(t)$ (which accounts for both prepayments and defaults), and experiences a principal-loss rate of $d(t)$ measured off the remaining balance with all occurring continuously. Then Equation (2) can be integrated by parts in a way similar to that shown in Levin [1998]:

$$P = 1 + E^{Q}\int_0^T [c(t) - r(t) - crOAS - d(t)] \exp\{-\int_0^t [r(\tau)+crOAS+\lambda(\tau)]d\tau\}\,dt \quad (3P)$$

If we carefully inspect this result, it can be interpreted as the pricing formula for a regular non-amortizing, non-defaulting bond that pays the coupon of $c + \lambda - d$ in the economy functioning under a $r + \lambda$ risk-free rate. Therefore, amortization rate $\lambda(t)$ is additive to both the paid rate and the discount rate, whereas the loss rate $d(t)$ is subtracted from the paid rate as we would expect.

A bond will be valued at par regardless of interest rates and amortization rate if it pays a perfect floater, $c = r + m$, indexed to the short rate r with the margin m exactly equal to the expected loss rate d plus *crOAS*. This is true for the majority of ABS deals at origination. Once the expectation of losses or liquidity (reflected in *crOAS*) changes, the bond will be valued at a premium or discount which becomes a function of interest rates and amortization.

The loss stream itself is valued at

$$L = E^{Q}\left[\int_0^T d(t)\exp\{-\int_0^t [r(\tau)+crOAS+\lambda(\tau)]d\tau\}\,dt\right] \quad (3L)$$

If all the rates are constants and the instrument is in perpetuity, then the integrals are taken easily:

$$P = \frac{\lambda + c - d}{\lambda + r + \alpha rOAS}, \quad L = \frac{d}{\lambda + r + \alpha rOAS}$$

VALUATION MODEL RECIPES

The number of mathematical states that describes interest rates, home prices, cash flows, and the credit model is rather high which makes random simulations the only candidate for a Credit OAS pricing scheme. The number of random forces is limited to those driving interest rates, the HPA diffusion, and the HPA jumps. For example, the use of a single-factor term structure model will ultimately require the concurrent simulation of three random shocks. Because many ABS have a complex structure and complex cash flow rules, and are backed by heterogeneous collateral groups, realistic applications of the Credit OAS method severely limit the number of paths. We have found a few recipes that are tested and have been proven useful, but many others have been tried with mixed results.

Path "Fudging" instead of antithetic reflection. Using rudimentary theoretical information, such as the knowledge of market discount factors, artificial adjustments can be added to the random processes to "center" them on correct values. Consider, for example, a Monte Carlo method for interest rate derivatives. Adding a "fudge" factor to the short-rate paths to ensure exact values for the discount factors is a simple and effective technique. Similarly, adjustments can be made to long rates and shocks generated for the HPA diffusion and jumps.

We consider this method superior to traditional antithetic reflection because it does not double the number of random paths. Furthermore, we can correct the inaccuracy in an explicit way related to the financial application. In contrast, antithetic reflection of interest rate paths does not ensure exact discounting.

Ortho-normalization of random shocks. If shocks are scaled to 1.0 and made orthogonal to each other, we can ensure that random paths follow the correct volatility patterns, a critical element in option pricing. This technique can be classified as moment matching (Glasserman [2004]). We first start with random numbers and then apply the Gram-Schmidt method to transform them into properly scaled, serially independent samples. All the second moments, variances and covariances, exactly match theoretical levels. Although the shocks used to generate interest rates, the HPA diffusion, and the HPA jumps are uncorrelated, the HPA process is, of course, linked to the interest rate process by construction. Our simple MBS pass-though tests show that ortho-normalization doubles Monte Carlo accuracy and stabilizes conversion.

Seed shuffling. If all that is needed is an assessment of the expected loss or price of a large loan portfolio (not a collateralized mortgage obligation, or CMO), there exists a very simple and effective method—use a few Monte Carlo paths per position and start each position's run from random seed. It turns out that from a portfolio standpoint this approach is equivalent in accuracy to many independent Monte Carlo runs. Efficiency depends on the homogeneity of the loans. Imagine, for example, that the collateral pool is composed of 1,000 perfectly identical loans. Running two random paths per loan, seeded randomly, is equivalent to running 2,000 random paths from a portfolio standpoint. In contrast, running two random but identical paths per loan is no different than running only two paths for the portfolio. Even if the loans are somewhat heterogeneous, using random seeding instead of same seeding will be just as accurate for each loan and more accurate for the portfolio. Furthermore, if the collateral is composed of one million loans (rather than one thousand), we might even extend our faith in Monte Carlo and instead of running two randomly seeded paths per each loan, apply them to a few thousand randomly chosen loans.

In Exhibit 3 we demonstrate that using a few *different* paths per loan allows us to assess the price of losses rather accurately for both fixed-rate and adjustable-rate (ARM) groups of the subprime CW0708 deal issued by Countrywide. In contrast, using a few *identical* paths for each loan is less accurate.

The few-paths random-seed method benefits from error diversification, much like investing in many independent stocks. It also suggests that the typical dream of senior management to have every position priced consistently against the same set of paths will likely reduce the accuracy in risk measurement, without benefits. Unless positions need to be accurately valued versus each other (e.g., asset versus hedge or specified pool versus TBA), using same paths for every position is not advantageous. When measuring duration, convexity, and other greeks, we must keep the initial seed unchanged, but should change it when moving from one position to another.

Loan clustering. Running a structured deal using a loan-level Monte Carlo simulation is not a practical approach. Even if the best assessment of losses and their distribution requires loan-level modeling, a direct stochastic simulation of such a model is prohibitively long. Instead, we can employ a single market scenario to compute prepayments, default rate, and losses for every loan. Using these measures (scores) we can then cluster loans so that they are grouped into a few internally homogeneous clusters. In a very simple case, we can use the loan loss score and create two loan groups: active and passive.

Clustering is a field that lies between computer science and statistics. It employs both established theoretical facts and proven numerical processes (typically, iterations). For example, the optimal solution is such that each loan is closer to its own center (centroid) than to other centers. Our preliminary study, which is beyond the scope of this article, proves that using only two loan groups clustered by a loss score derived from a simple static analysis is a reasonable method. For example, if we ran a full Monte Carlo with this two-group surrogate, we could assess the collateral pool's loss distribution with a suitable accuracy rate of 3% to 5%. A three-group formation would produce a further improvement in accuracy, albeit a rather modest one. In contrast, using only one cluster (i.e., the single weighted-average repline) leads to an unsatisfactorily large error of 20% to 25%.

WHERE DOES THE RISK-NEUTRAL DISTRIBUTION OF LOSSES COME FROM?

Implied Versus Modeled Loss Distribution

A strong feasibility test for our method is its ability to recover the market-implied distribution of losses. As previously mentioned, in order to assess the damage to any ABS, the entire probability distribution of collateral has to be known. We continue to use the CW0708 subprime deal as an example of the analysis.

First, we attempt to recover the loss distribution using tranche prices via the Gauthier [2003] scenario grid method. We work with nine different classes, M1 through M9, priced by the market in May 2007. We then construct 20 credit scenarios, ranging from ultimately optimistic (no new defaults) to unreasonably pessimistic. Our assumptions about prepayment scale and default and loss severity rates for each scenario are somewhat arbitrary but appeal to financial intuition. For example, we couple lower prepayment scales with higher default rates; the exact definition of the credit scenario grid should not materially affect our results. For each credit scenario, we run a regular OAS model that incorporates stochastic interest rates and prepayments, but we apply constant default and loss severity rates. Because we account for losses by altering cash flows, the OAS level we employ should match that of agency MBS, adjusted for the appropriate liquidity difference. At the end of this step, we compute the entire scenario-by-tranche pricing matrix which is shown in Exhibit 4.

Next, we interpret credit scenarios as random with probabilities that total 100%. Therefore, the price of each tranche should equal the probability-weighted scenario prices. Finally, we find an optimal probability distribution of the credit scenarios so that the computed prices approximate market quotes as closely as possible. Instead of assigning each scenario an independent probability value, we prefer to parameterize the distribution using the Vasicek loss model [1989, 1991]. This is a two-parameter skewed distribution derived to depict default rate probability in an infinite pool of identical loans. Exhibit 4 shows the optimal probability. Most tranches are reasonably close to actual market prices (the average root mean squared error is 0.43%).

So far, our exercise has not involved the method of Credit OAS because we simply inferred losses from bond prices. Let us now apply the Credit OAS method and measure the loss distribution it generates. We are interested, moreover, in changing the characteristics of the HPA model in such a way as to approximate the distribution shown in Exhibit 4. Remember that parameters $D(0)$ (short-term forecast) and $D(\infty)$ (long-term level) can be viewed as tuning parameters as well as factors of risk. For simplicity, we limit this feasibility study to finding $D(\infty)$ only.

Exhibit 5 depicts the distribution of losses arising from different $D(\infty)$. Observe that had we lowered the long-term level of HPA to about 2%, the distribution of losses would have approached the implied distribution. This is an important result which attests to the validity of our approach. In short, it means that at the time of analysis a two-factor stochastic world presented an adequate market view of the loss distribution. We could also have included the short-term tuning factor, $D(0)$; the volatility of the jump component, σ_{HPA}; or any other parameter in the HPA Equation (1) to match the implied loss distribution.

Having established the risk-neutral correction, we can use it for other ABS deals. At first glance, a 2% long-term HPA rate may seem low and almost impractical. But upon closer inspection, it does not appear to be so outrageous.

EXHIBIT 3
Monte Carlo Convergence for CW0708

	Fixed Rate Loans		Adjustable Rate Loans	
Accurate	6.928		6.107	
	Random Seed	Same Seed	Random Seed	Same Seed
100 paths per loan	6.933	6.901	6.111	6.074
20 paths per loan	6.977	6.857	6.122	6.069
10 paths per loan	6.983	6.727	6.139	5.936
2 paths per loan	6.876	5.495	6.068	5.301

First, we expect the risk-neutral HPA assumption to be worse than one based on empirical judgment; it must include the price of risk, which we will quantify in the next section. Second, our physical HPA model can be viewed as rather optimistic because it is based on OFHEO historical data which include only the conforming loan sector and exclude foreclosure transactions, thus inducing an upward bias. Third, we should attribute some of the 2007 market pessimism to the short-term forecasts which we left unaltered in this case study. Had we sought the best combination of $D(0)$ and $D(\infty)$, we would certainly have used a more optimistic $D(\infty)$. The tuning recommendations, however, change with the market (see our case study in the last section).

Capital Charge and Loss Distribution

In order to quantify the risk-neutral conditions and link them to a financial rationale, let us consider the positions of loan insurer (limited insurance coverage), GSE (unlimited insurance), and CDS protection seller. In any case, a premium, p, should be charged to make the business profitable. Without a doubt, the insurer must charge more than the expectation of losses, μ, in the real world. By definition, insurance provides protection against worse than average events, so capital, c, must be maintained to back the obligation. This capital must remain very liquid and therefore can be expected to earn only the risk-free rate, r, which is much lower than the return-on-capital target, R. Let us assume that the standard deviation of losses is σ and that the insurer must keep capital sufficient to cover, $k\sigma$, in addition to μ.

The expected total return will reflect the (+) risk-free return on capital, (+) guarantee fee, and (–) expected claim μ:

$$R = \frac{cr + p - \mu}{c} = r + \frac{p - \mu}{c} \tag{4}$$

under the constraint that the capital is large enough to cover a worst-case loss,

$$c \geq \mu + k\sigma - p \tag{5}$$

From (4), the insurer is interested in charging $p \geq \mu$ and minimizing the amount of capital that must be held. From inequality (5), the minimal level of capital is $c = \mu + k\sigma - p$. If $\sigma = 0$, no capital is required, assuming $p \geq \mu$. Therefore, we can present the capital requirement as a multiple of the standard deviation, $c = x\sigma$. Then, replacing $p - \mu$ with $(x - k)\sigma$ in formula (4), we get

$$R = r + \frac{k - x}{x} \tag{6}$$

The sequence of computations using formulas (4) through (6) is as follows. First, we select the underlying financial instrument, either the agency guarantee, mortgage insurance, or CDS. Assume that R is the desired expected ROE. Equation (6) defines the capital scale, x,

$$x = \frac{k}{1 + R - r} \tag{7}$$

Let us find the premium rate, p, which results from this consideration. Substituting $c = x\sigma$ into inequality (5) and treating it as an equality, we solve for p,

$$p = \mu + k\sigma \frac{R - r}{1 + R - r} \tag{8}$$

EXHIBIT 4
Implied Default Model (IDM)

Scenario	Default	Severity	Prepay	Delinquency	Recovery	Cumulative Loss %	Optimal Probability	M1	M2	…	M9
	Scales in Order of Increasing Losses							Tranche OAS Prices			
1	0.00	0.00	1.00	0.00	1.0	0.00%	0.00%	100.495	100.586	…	108.690
2	0.25	0.94	0.95	0.50	1.0	1.46%	1.03%	100.469	100.560	…	108.632
3	0.50	0.96	0.90	0.60	1.0	2.97%	9.02%	100.441	100.531	…	108.470
4	0.75	0.98	0.85	0.80	1.0	4.55%	19.73%	100.445	100.532	…	107.271
5	1.00	1.00	0.80	1.00	1.0	6.20%	16.30%	100.433	100.477	…	106.907
6	1.10	1.02	0.78	1.05	1.0	7.01%	8.91%	100.642	100.751	…	113.392
7	1.20	1.04	0.75	1.10	1.0	7.85%	8.16%	100.522	100.765	…	121.313
8	1.30	1.06	0.73	1.15	1.0	8.73%	7.21%	100.523	100.735	…	122.864
9	1.40	1.08	0.70	1.20	1.0	9.64%	6.18%	100.535	100.757	…	101.670
10	1.50	1.10	0.68	1.25	1.0	10.60%	5.17%	100.549	100.791	…	59.592
11	1.60	1.12	0.65	1.30	1.0	11.60%	4.22%	100.565	100.834	…	38.608
12	1.70	1.14	0.63	1.35	1.0	12.64%	3.38%	100.583	100.887	…	30.724
13	1.80	1.16	0.60	1.40	1.0	13.72%	2.67%	100.603	100.950	…	27.710
14	1.90	1.18	0.58	1.45	1.0	14.84%	2.07%	100.629	101.022	…	25.870
15	2.00	1.20	0.55	1.50	1.0	16.00%	3.66%	100.665	101.096	…	24.480
16	2.50	1.22	0.53	1.55	1.0	18.65%	1.90%	100.772	95.375	…	21.238
17	3.00	1.24	0.50	1.60	1.0	20.99%	0.33%	100.593	52.815	…	19.195
18	3.50	1.26	0.48	1.65	1.0	23.07%	0.05%	85.522	23.295	…	17.679
19	4.00	1.28	0.45	1.70	1.0	24.94%	0.01%	50.708	19.500	…	16.474
20	5.00	1.30	0.40	1.80	1.0	27.71%	0.00%	20.263	16.820	…	14.798
					Probability Weighted	**7.84%**	100.00%	**100.506**	**100.370**	…	**93.042**
					Quoted Market Price			**99.973**	**99.860**	…	**93.167**
	per bond mispricing	**0.43**			**Mispricing**			**0.532**	**0.510**	…	**-0.125**

minimize **optimize**

Formula (8) is the insurance premium that minimizes the amount of the insurer's capital, given return target R, insurance limit k, and the loss distribution pair (μ, σ). The result is stated in the form of *expected loss plus capital charge*. All measurements should first be carried as present values, then translated into an annual fee dividing by the annuity known as the IO multiple. The annualized p is known as the guarantee fee, insurance premium, or CDS rate.

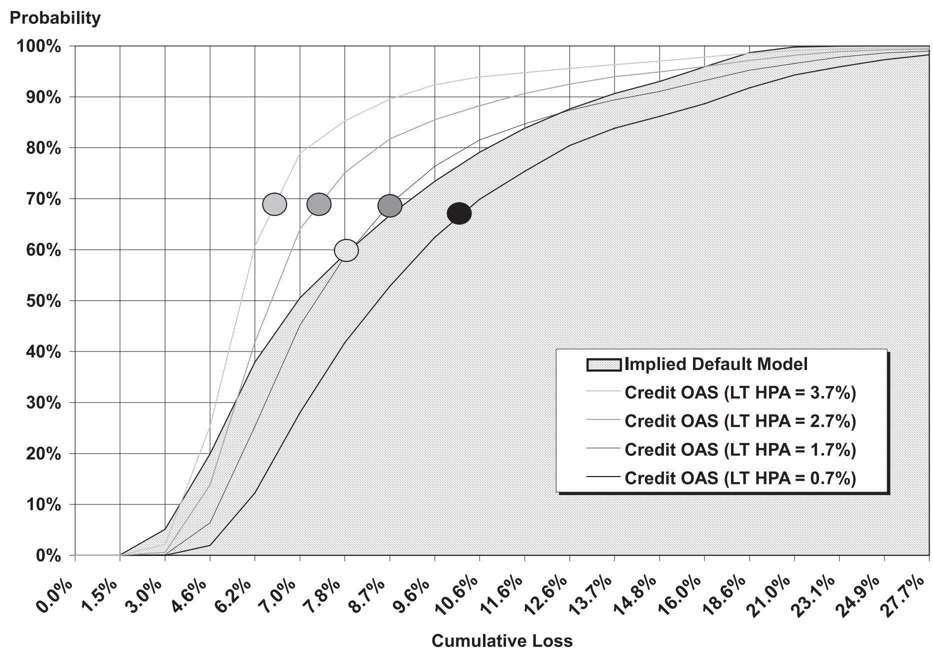
Example:
Inputs: μ = 5 bps/yr, σ = 5 bps/yr, R = 25%, r = 5%, k = 6.
Outputs: x = 5, c = 25 bps/yr, and p = 10 bps/yr, 50% of which is the expected loss and another 50% is the capital charge.

Some observations follow immediately:

- Capital is proportional to k, the insurance scale. The insurance premium, p, is linear in k.
- If the desired return rate, R, grows, then capital is initially affected only minimally. The insurance premium, p, is almost linear in R. When R goes to infinity, however, required capital vanishes and the insurance fee converges to $\mu + k\sigma$; that is, it alone completely covers worst-case losses.
- The price of a unit of risk is $\pi = k\frac{R-r}{1+R-r}$. It equals 1 in the preceding example. Price of risk depends on the risk-free rate, return target, and insurance confidence. The level of insurance confidence will differentiate insurance policies.

How does this economic reasoning relate to risk-neutral market conditions? In the mortgage insurance business (the *underlying*), equilibrium prices are formed when insurers and clients agree on R and k, thereby defining the price of risk, π. This defines the right side of formula (8) which becomes the risk-neutral expectation of losses. In this case, ABS investments and CDS protections can be viewed as derivatives and valued using the capital asset pricing model. Eventually, the market players who take long and short positions agree on the price of risk, π, although each makes his own estimate of R related to the exposure of his position.

EXHIBIT 5
Risk-Neutralization of Loss Distribution in Credit OAS (May 2007)



One important conclusion is that if an analytical approach is needed to assess both the expected loss and the capital charge, then the Credit OAS should deliver a richer information output set than just the present value of losses. In particular, the formulas in this section require computing the path-wise standard deviation of losses. Their direct use may be necessary to find the economic value of illiquid financial instruments or when it is difficult to find an established benchmark market. If the conditions of risk neutrality can be easily inferred from prices of traded instruments, explicit knowledge of the values of μ, σ, k, and so on is not required; it is enough to shift the distribution of modeled market factors, like the HPA, to match the price of losses shown in (8). This is the process we followed in the previous section in order to find the prices of the CW0708 tranches.

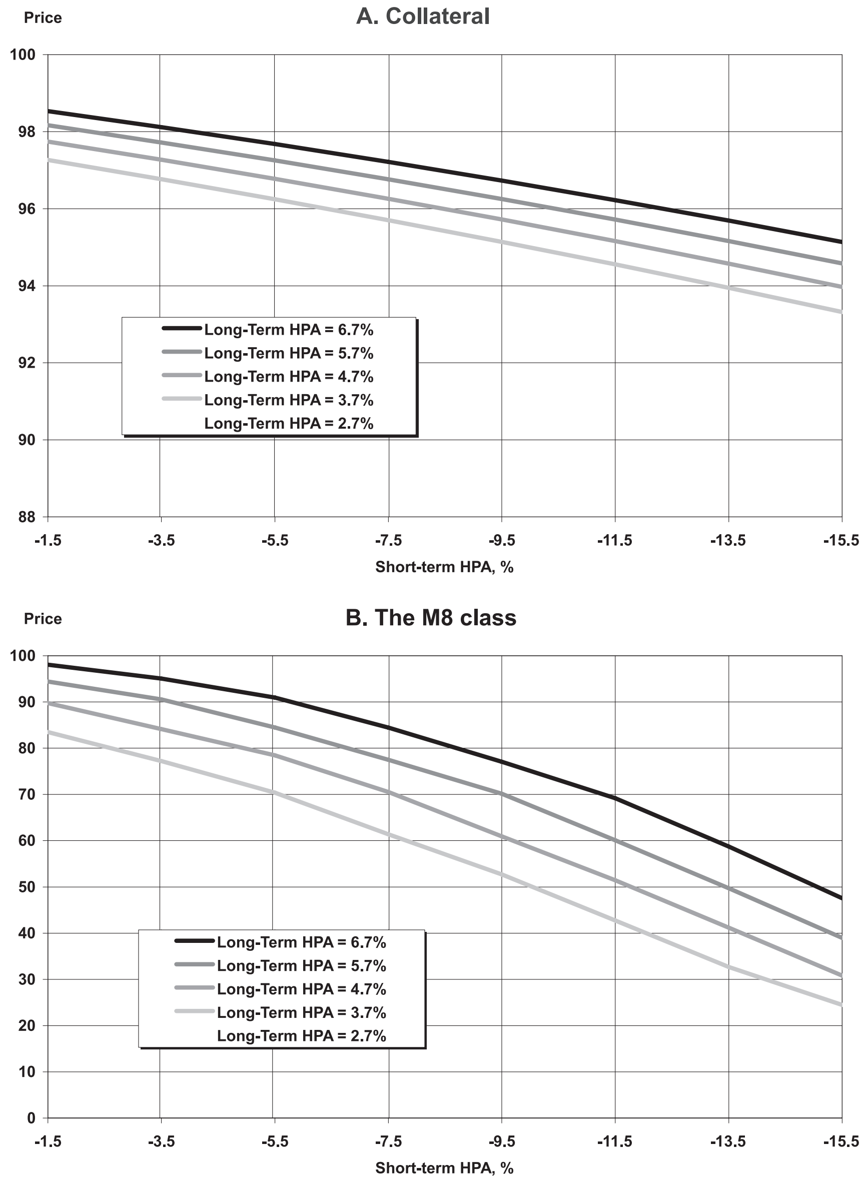
HPA Derivative Markets

One developing source of home-price risk neutrality is the real estate derivatives market. In 2006, the Chicago Mercantile Exchange (CME) launched the trading of futures and options using the S&P/Case-Shiller® Home Price Index. The index employs the repeat-sales method and is published monthly; the CME futures mature quarterly. The market includes a U.S. composite and ten cities.

In September 2007, another market was established by way of the Radar Logic Residential Property Index™ (RPX), which offers daily prices computed on a per-square-foot basis. This market started with the trading of option-free contracts (total return swaps) having a maturity of up to five years for the U.S. composite and shorter maturities for the 25 metropolitan indices.

A number of major brokers are involved in both markets, along with ICAP, the interdealer broker facilitating RPX trading. The HPA markets (if they grow sufficiently large) can be used by mortgage insurers, GSEs, ABS investors, CDS protection sellers, and homebuilders to hedge exposure to home prices as well as to speculate. In order to execute hedge strategies, an HPA hedge ratio has to be calculated. Similar to the case of interest rate risk, the Credit OAS method should deliver all the relevant greeks. In particular, using our modeling views on HPA, the instrument's exposure to $D(0)$ and $D(\infty)$ must be assessed and employed for taking an HPA hedge position. For example, our model shows that losses in a typical subprime pool have a duration of five to ten years to the HPA level and about one-third of that to the short-term HPA. This means that the present value of losses can increase or decrease by 5% to 10% for each percent of the HPA level. The exposure is positively convex and reflects the optionality of the borrower's default, that is, duration grows with losses. In common ABS credit enhancement structures, this dependence is highly nonlinear.

EXHIBIT 6
Value Surfaces of the SAS06BC4 Deal (September 2007)



The short-term HPA rate is an instantaneous annualized rate. It is not equal to the first-year HPA rate, which is composed of 63% short-term HPA and 37% long-term HPA. The second-year HPA is composed of 23% short-term HPA and 77% long-term HPA, and so on.

A PRACTICAL ABS CASE STUDY (ILLIQUID MARKET)

We complete the article with a case study of the SAS06BC4 subprime deal. The date of the analysis is September 28, 2007, by which time the deal had accumulated delinquencies of 5% and severe delinquencies of 9.5%. We analyze four tranches—A5, M2, M5, and M8—with differing credit protection as well as an analysis of the deal's collateral pool. The protection for each tranche had risen since origination primarily as a function of the very low level of already accumulated collateral losses (0.22%). Fairly large losses are obviously expected to occur in the future given the impaired pool composition and the fall or stagnation in home prices. Actual pricing quotes were available for all tranches, although these prices should

Exhibit 7
Valuation Comparison Using Regular OAS and Credit OAS

Tranche		Regular OAS Method			Credit OAS Method			
	Price	*OAS*	*OAD*	*OAC*	*crOAS*	*crOAD*	*crOAC*	*PV(loss)*
A5	92.265	312	0.7	0.1	254	1.6	-0.7	0
M2	75.425	737	2.1	0.4	511	7.8	-5.0	2.1
M5	65.443	876	3.3	0.6	378	20.9	-9.8	17.3
M8	37.126	2,030	3.8	1.6	480	47.9	7.0	57.6

be viewed with caution due to the illiquidity of the market. Nevertheless, we show how the Credit OAS method can help to analyze the deal from many practical angles.

Valuation Surface

We start by computing the values of the instruments using a *crOAS* of zero. This approach cannot lead us to the actual prices because we are missing the liquidity spread,[5] but it lets us construct a value surface in the space of two tuning factors in the HPA model—the long-term diffusion, $D(\infty)$, and the initial diffusion level, $D(0)$. Note that for the same values, tuning factor $D(\infty)$ should generally be stronger than $D(0)$ because the diffusion term transitions rather quickly in the model. At the same time, the short-term HPA rate is often more uncertain; for example, balloon-bursting assumptions made by different analysts and firms varied widely in fall 2007.

Exhibit 6 depicts the zero *crOAS* prices of the instruments in the space of two HPA tuning factors (we depict and compare only the collateral and the M8 tranche). The collateral value changes smoothly and exhibits some negative convexity with respect to each of the HPA factors. This observation agrees with the fact that the borrower's default is an option; positive convexity might have pointed to a flaw in the model. It also means that, had we resorted to the static valuation method commonly employed for credit analysis, we would have understated collateral losses. The value of the M8 tranche resembles a short position in digital options; that is, it is negatively convex when the HPA is high (option is out of the money) and positively convex when the HPA is low (option is in the money). This is the direct consequence of a typical credit enhancement within an ABS capital structure. If ABS investors liked to hedge this exposure, they might want to see the CME and ICAP to incept digital home price options.

Getting to the Right Point

Now, we try to pinpoint the combination of $D(0)$ and $D(\infty)$ that best approximates market prices for the M8 to A5 classes. It appears that we are less successful in performing this calibration exercise than we were for the May 2007 analysis of the CW0708 deal discussed earlier. For example, we can easily match depressed prices for the junior tranches (M5, M8), but a reasonable set of home price tunings is not available to justify the pricing quotes of the senior tranches (A5 quoted at 92.265 and M2 quoted at 75.425); both seem to maintain deeper actual protection than the market quotes indicate.

A plausible financial explanation could stem from the liquidity spread. Valuation using *crOAS* is bound to a properly selected, similarly liquid benchmark. In the illiquid ABS market, we may assume that part of the pricing discount reflects an enormous bid-ask spread. Thus, we can use the HPA tuning assumptions— –4.7% of the long-term HPA rate and –11.5% of the short-term HPA rate—that will result in reasonable liquidity spreads. Given these assumptions, the expected collateral loss is 12.4% in present value terms; the expected losses for the tranches are shown in Exhibit 7. The rest of the pricing discounts will be absorbed by *crOAS* in recognition of impaired liquidity. We compare the main valuation measures using two alternative methods: the regular OAS method and Credit OAS. To compile the regular OAS results, we run the same model with a zero loss severity to ensure that total collateral amortization, both voluntary (turnover, refinancing) and involuntary (default), is similar to that of the Credit OAS method.

The levels of *crOAS* appear to be wide, but plausible, for the illiquid market (A5 has better liquidity). The most stunning difference between the two methods is the projected interest rate sensitivity in that a duration of 50 years is intimidating and unlikely can be proven by daily prices.

It is merely a result of the strong relationship between interest rates and home prices ($k \approx 2$) in the HPA Equation (1). The results agree, however, with the notion of lowering interest rates as a recipe to curtail the credit crisis. Whatever the true OAD is, our analysis suggests it can vary widely with HPA models and is unlikely to remain close to the traditional measure.

CONCLUSION

The concept of Credit OAS revolves around coupled simulations of interest rates and home prices and is imperative to rationally explain prices of traded ABS, CDS, and loan protections. The Credit OAS approach requires a risk-neutral stochastic model of home prices, a model of defaults and losses (theoretical or empirical), and a rigorous and efficient valuation scheme. The risk-neutral assumptions can be derived from concurrently observed prices of ABS tranches throughout the credit structure. An attractive alternative is to provide theoretical premiums for GSE loan guarantees and mortgage insurance based on expected losses and capital requirements. Once the risk-neutral evolution of home prices is established, it can be used to price other credit-sensitive instruments (derivatives).

The model can be employed for both liquid and illiquid markets. Even distressed market prices can be explained by a combination of modeled losses (under risk-neutral interest rates and home prices) and a properly selected credit OAS level (i.e., liquidity spread). The greeks, however, will depend strongly on the modeling details (such as the link between interest rates and home prices), vary widely from model to model, and likely differ from the measures computed using the traditional OAS approach.

ENDNOTES

The authors wish to thank Will Searle and Daniel Swanson for their software implementation and integration work. The LDM modeling work and its validation is due to Kyle Lundstedt and Anne Ching.

[1]Part of the agency guarantee premium is related to the liquidity advantage of the agency MBS.

[2]Our HPA model includes seasonality which is not shown in Exhibit 2.

[3]The SATO can point to credit quality not observed in other loan characteristics.

[4]At origination, all the loans are normally considered current.

[5]The valuation surface can be constructed for any realistic *crOAS* level.

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